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# Unsteady flow simulation of water drainage in open-graded asphalt mixtures

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**Abstract**

Laboratory tests or measures on the field are widely carried out in order to evaluate the drainage capability of pavement. Such an approach is in general not efficient and it shows poor significance. In this paper we propose a novel method, very effective for mix design, based on simulation of unsteady-flow of water through open-graded mixture. In particular we have modelled internal microstructures of an open-graded asphalt sample, positioning any aggregate particle and creating the bitumen film around each of them. The unsteady-flow of water inside the sample is simulated to evaluate the expected permeability for different specimens and compaction.



Keywords: Lattice-Boltzmann, RSA, flow simulation, open-graded asphalt mixtures

## 1. Introduction

One of the most important factors controlling pavement performance is the ability of open graded mixtures to prevent water from remaining within the pavement system. This ability is commonly called permeability. This is related to the capability to transmit fluids through voids when subjected to hydraulic loads. It is very difficult to predict the pavement permeability because it is a function of many variables like the different mix constituents, the compaction and other geometrical properties of the pavement. The traditional approach to evaluate the expected drainage capability of pavement is based on laboratory tests using hydraulic permeameters or measures on the field. The main advantage of the field permeameters is that the test is non-destructive. However it has to be noted that the flow is possible in any direction, so that the resulting permeability is not the one dimensional Darcy permeability, the determination of the degree of saturation is often inaccurate, it is difficult to evaluate the exact thickness and area of the pavement layers, and the number of layers in which fluid flows cannot be often determined in a reliable way. Experimental instruments and setups have been developed for measuring HMA

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permeability in the laboratory. Laboratory setups overcome some of the mentioned points of weakness of the field tests. In particular consistent degree of saturation can be assured during the laboratory test. The permeability, according to Darcy law, can be measured using two setups: the falling-head and constant-head. These setups differ mainly in the way in which the head is applied during testing (constant-head or falling-head); moreover the flexible triaxial compression chamber (ASTM D5084-90 1999) or rigid wall permeameter are used (ASTM D2434-68 2000). In the last years numerous researchers have attempted to calculate permeability through novel models. These models have been also applied to evaluate the drainage capability of the pavement. In general the permeability models are categorized as analytical, probabilistic, numerical, and morphological [1, 15]. The Analytical Models predict permeability of porous materials from some measures of voids content. Bear [2] reviewed a group of models that have been used to study flow through porous media. Rajani [20] summarizes these models in four different analytical approaches: fissure, capillary tubes, resistance to flow and hydraulic radius. Each of these approaches are based on different equations. Masad [15] proposed a general equation to represent evaluation of pavement permeability. In this equation the permeability is a function of many variables like voids content or porosity, distribution of voids, and average diameter of the particles of the porous media. This equation is based on simplified assumption of shape and distribution of voids as well as grading and shape of aggregate. Under these assumptions the equation describes the internal structure by average parameters such as voids content, average particle size, and specific surface area of aggregate particles. However, since HMA includes a wide range of aggregate sizes, it would be difficult to represent the effect of grading only by an average particle size, making often the predictions not reliable. The Probabilistic Models are based on the evaluation of the probability that there is hydraulic interconnection among voids [19, 22]. In general, these models utilize elaborate procedures to describe the different distributions of voids in porous materials. In addition these models assume some simplified hypothesis on the probabilistic distributions of voids and their connectivity, neglecting the constitutive scale. The solution is acceptable at a macroscale but the level of detail at the microscale is poor. The Numerical models [21, 25] calculate the permeability of porous materials by solving the fluid flow equations numerically. These equations generally include the continuity equation and the momentum equations. Different computational techniques are used to solve these equations, typically the finite difference and finite elements models. In both cases the solving equations are derived from Navier - Stokes equations. The equations are solved through a linear combination of the basic functions defined for each infinitesimal element of the whole domain. The knowledge of the internal microstructure of the material is required and these models are very onerous from a computational and economical point of view. The Morphological Analysis Models take into account the microscopic geometry of voids [9, 12]. They are based on an equivalent random network of cylinders or spheres to model the microstructure and geometry of voids. This approach requires expensive and onerous experimental procedures that can also alter the void distribution. In this context we propose a novel method based on simulation of unsteady-flow of water through open-graded mixture. This method can be considered an integration of Numerical and Morphological Analysis approach. This novel approach reduces computational complexity of the full morphological models and increases the reliability of numerical models. The overall objective of this study is to develop a new mathematical procedure to predict pavement permeability in order to support the design and to optimize the mixture. The results of simulation models are validated using experimental tests (hydraulic permeameter) and analytical predictions.

## 2. Numerical Models

The proposed approach is based on two different numerical models, that are integrated in a cascade. The first one is the model for the asphalt mix samples generation, it is based on Random-sequential adsorption (RSA). In this model a set of spherical particles is randomly extracted, so that the distribution of the spheres diameters is coherent to the distribution of aggregate grade and then each particle is positioned in the geometrical domain of the sample following RSA approach [4]. The second one is the model for the unsteady flow of water infiltration

based on Lattice-Boltzmann (LB) method. This method approximates the continuous Boltzmann equation by discretizing a physical space with lattice nodes, where each node corresponds to a void or to a solid, and the velocity depends on a set of microscopic velocity vectors. Fluid particles travel on these nodes following the main LB equations, that satisfy the Navier-Stokes equations [11]. The algorithms are developed in C++ language. In the following chapters we explain the basic theories..

### *2.1. Numerical Generation*

According to Benedetto [4], the model is based on the micro-simulation of the physical structure of the mixture. Numerical samples of open graded mixes are generated from mix-design inputs in the form of Marshall test samples. Starting from a fixed distribution of aggregate sizes it is possible to generate an amount of samples that are different for the casual aggregates arrangement. Here a random sequence is adopted to generate particles dimensions and to find their position in the space inside the sample domain. Finally each particle is covered by a film of bitumen. This simulation model is based on Random-sequential adsorption (RSA). RSA is a simple but fundamental problem in statistical physics. Objects are added randomly, one at a time, to a d-dimensional space. They must not overlap with previously added objects. As the process of adding objects is proceeding it becomes more and more difficult to find regions where the objects can find a new place. When no further additions are possible the process is reaching automatically the "jamming limit" [17]. Assuming that the asphalt production is a random process, the RSA can be accepted as a good approximation of the mixture forming. The single particle of aggregate is approximated here by a sphere. In general it is evident that the spherical approximation is not always very realistic for this purpose, but in the case of porous mix with homogeneous grading this approximation has been validated at a first stage [4]. The diameter of the sphere is an approximation of the specimen size. The spheres have diameters from a minimum value $\delta min$ to a maximum value $\delta max$. The method is here applied to simulate real porous hot-mix asphalt samples, with the dimensions and the shape of the traditional Marshall test sample (cylinder $\Phi$ 0.1016 m and 0.0635 m high). The RSA algorithm selects randomly one diameter (D) of a single sphere and one point (x,y,z) within the three-dimensional domain, that is the position of the center of the sphere. The selection of the diameter is made within the range of possible diameters ($\delta min \leq D \leq \delta max$). The set of selected diameters must accord to the distribution of aggregate sizes. At this scope the expected grading is divided in mutually exclusive but exhaustive classes (i.e. class i is from diameter Di to diameter Di+1). Within each class a set of diameters are extracted at random in a way that the final distribution of particles all over the classes accords to the real grading of aggregates. The simulation is stopped when the rate between solid, as sum of the particles volumes, and the volume of voids tends to the real rate. Finally the distribution of the diameters of the spheres is checked, to verify the consistency of the simulation respect to the real grading.
Two different approaches have been adopted. A complete three dimensional approach and a simplified two-dimensional approach. The sample are shown in figure 1:

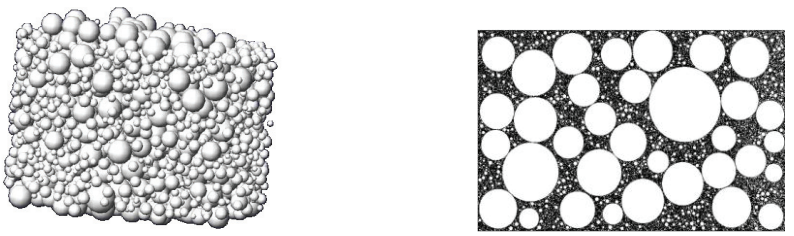

Fig. 1. (a) full scale three-dimensional simulated sample using RSA, (b) vertical section of simulated sample using RSA

It has been verified [4] that this second approach (2D) gives results that are strictly correlated to the 3D approach and it can be considered absolutely more efficient, under a computational point of view, it a full and accurate

generation of the real sample is not required. According to a single aggregate grading and a single bitumen content it is possible to generate how many different samples how the different random seeds are. This casual procedure simulates the real procedure of sample making. It is well known that the laboratory procedure to make samples produces, from the same initial conditions, different results, also under rigorous standards and methods. This is the reason why, in the laboratory tests, a number of samples (generally four) is required to calculate the average values of mechanical or hydraulic properties. For obvious reasons of time and cost this number of samples is always very limited. In the mix design step, the numerical simulation makes it possible to generate a great amount of virtual samples. Over this great amount of samples it is possible to extract more stable and representative averages, following a Monte Carlo procedure [18].

### *2.2. Simulation of Unsteady Flow of Water inside the Sample*

Lattice – Boltzmann method (LB) is a class of computational fluid dynamics methods (CFD) for fluid simulation. It has evolved from the theory of Lattice Gas Automata (LGA) [23]. It was first introduced by McNamara and Zanetti [16] to eliminate the statistical noise in the LGA. Since then, it has been implemented and improved by various researchers in a variety of disciplines [10, 13]. The basic idea of LB method comes from the kinetic theory of fluids. LB method considers a representative volume of fluid (a node in the lattice) composed of particles that are defined in terms of the velocity distribution function, F (x,t) [14]. This function is a discrete function that has Q number of components, where Q is the number of microscopic velocity directions (ei) around a lattice node. Each component of the function (Fi, i= 1, 2 …Q) represents a fraction of the total number of particles at each node with a microscopic velocity $e_i$ (Fig. 2).

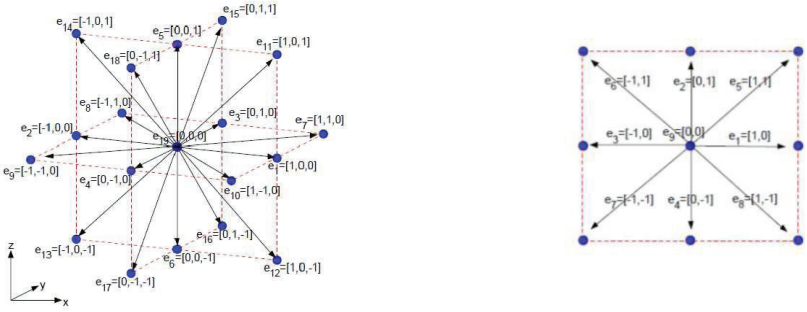


Fig. 2. (a) three-dimensional lattice velocity directions, (b) two-dimensional lattice velocity directions

The fluid particles travel on the lattice nodes according to the distribution function which determines the macroscopic flow parameters, i.e., density and macroscopic velocity of fluid at each lattice node.

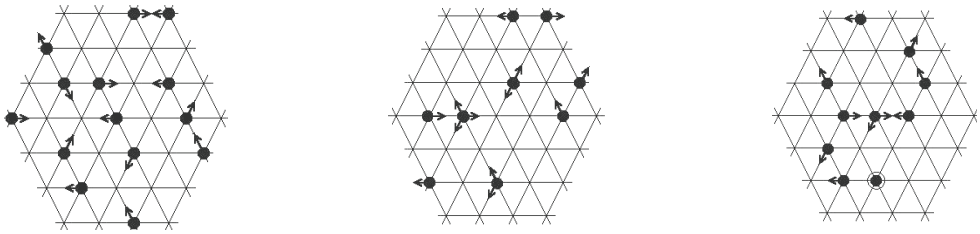

Fig. 3. (a), (b), (c) fluid particles travel on the lattice nodes

Various LB models exist for numerical computation of various fluid flow scenarios, where each model has different way of characterizing microscopic movement of the fluid particles. The LB models are usually denoted

as DxQy where x and y corresponds to the number of dimensions and number of microscopic velocity directions (ei). In this case we will refer to the model D2Q9.

Lattice–Boltzmann method, as described previously, approximates the continuous Boltzmann equation. Generalized form of this equation is given as [7]:

$$\frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} + F\frac{\partial f}{\partial u} = \Omega(f) \tag{1}$$

where:

f: distribution function;

u: velocity;

F: body force per unit at mass;

Ω( f ): collision function.

The complexity of Boltzmann equation, especially over complex domain, makes it impractical to be solved directly for macroscopic flow problems. To simplify it LB method is applied. The discrete form of Boltzmann equation, obtained by discretizing physical space into a set of uniformly spaced nodes (lattice) and velocity space into a set of microscopic velocity vectors, can be written for each discrete (microscopic) velocity direction (ei), as follows [8]:

$$F_i(x + e_i, t + 1) = F_i(x, t) + \Omega_i - B_F \tag{2}$$

where:

$F_i$: (discrete) particle distribution function at lattice node x, at the time t;

$B_F$: body force.

The collision function, Ωi , can be simplified by assuming that the distribution functions relax to their equilibrium state at a constant rate (also called Bhatnagar-Gross-Krook (BGK) approximation [5]. Then the collision function takes the following form:

$$\Omega_i = -\frac{F_i - F_i^{eq}}{\tau} \tag{3}$$

where:

$F^{eq}$: equilibrium distribution function ;

τ: relaxation time which is a function of viscosity of the fluid.

Equilibrium distribution functions for different models were derived by He and Luo [8]. The function for D2Q9 model is given in the following form:

$$F_i^{eq} = w_i \rho \left[ 1 + \frac{e_i u}{c_S^2} + \frac{(e_i u)^2}{2c_S^4} - \frac{uu}{2c_S^2} \right] \tag{4}$$

where:

ρ: density;

u: macroscopic velocity of the node.

The relaxation time relates to viscosity of fluid (ν) as it follows:

$$\upsilon = c_S^2(\tau - 0{,}5) \tag{5}$$

where:

$c_s^2$: constant called lattice speed of sound.

The third component of the Boltzmann equation, Body Force (BF) assumes the following form:

$$B_F = -\frac{w_i}{c_S^2}(e_i \nabla p) \tag{6}$$

where:
$w_i$: weight factor for each direction around a node;
$e_i$: lattice velocity directions;
$\nabla$ p: pressure gradient.
The macroscopic properties, density and velocity, of the nodes are calculated using the following relations:

$$\rho = \sum_{i=1}^{Q} F_i \tag{7}$$

$$u = \frac{\sum_{i=1}^{Q} (F_i e_i)}{\rho} \tag{8}$$

## 3. Model Validation

The model proposed in this paper is validated through experimental tests and theoretical calculations. In particular experimental and analytical results from laboratory tests and Kozeny – Carman equation are compared to the model predictions.

### *3.1. Experimental test*

In this paper a laboratory test and equipment [3] are used to validate the model proposed. In particular, water is loaded by an hydraulic system from the head of a cylindrical vertical PVC pipe; the water flow is slowly loaded on the internal wall of the PVC pipe so as not to disturb the water surface over the Marshall sample, until the initial hydraulic load for testing is reached. If the test is a variable hydraulic load test, the water supply is closed, if the test is a constant hydraulic load test, the supply and the discharge valves are tuned, in order to maintain the water level at a fixed level. The equipment is shown in figure 4. A Marshall sample is inserted in a flexible resin clutch, close to the PVC pipe, and sealed at the circular boundaries by silicon. The water is loaded into the fixed test load (a constant hydraulic load or variable load could be performed). During a first initial phase, the infiltration begin, and the sample is not saturated. Permeability increases to a maximum value as the sample is saturated and the hydraulic transient ends. After this time, the test results are considered valid. The equipment allows to realize a standardized measurement of Darcy permeability, from the volume of the infiltrated water and from the hydraulic gradient.

Fig. 4. (a) some detail: insertion of the sample, (b) the sample sealed banging against the stops, (c) the sleeve is tightened with steel bands, (d) the piezometer is positioned in the seat at the elbow

### 3.2. Numerical test

The theoretical calculations are based on Kozeny – Carman equation [6, 24]. This is a semianalitical equation for the estimation of hydraulic conductivity of porous structures.

$$K = \frac{\gamma}{\mu} \frac{n_e^3}{bT^2} \frac{1}{S_a^2} \tag{9}$$

where:

K: hydraulic conductivity;

b: shape factor (2 for perfectly circular pore structure);

$S_a$: specific surface area;

$n_e$: effective porosity;

$\gamma$: unit weight of water;

$\mu$: viscosity of fluid;

T: tortuosity defined as the ratio of the total length of the path followed by a fluid particle to the shortest distance from the inflow to the outflow.

Following this approach we are able to calculate the hydraulic permeability. In particular, starting from the design requirements for HMA, it is possible to define the properties of the mixture, such as porosity and specific surface. It is then possible to calculate the values of tortuosity for the synthetic samples. Finally it is possible to calculate, by equation (9), the permeability for each sample and compare it to the laboratory outcomes.

### 3.3. Experimental Results

One grading curve has been selected from basaltic and silica – limestone aggregates, it is shown in table 1. Three different percentages of modified bitumen have been used to prepare the four Marshall samples for a total of 12 tests. The bitumen percentage are 5, 5.5 and 6%. The physical characteristics of samples are summarized in table 2. The hydraulic permeability of the 12 samples have been measured through permeameter Benedetto [3] after the saturation transient at different hydraulic loads (5 cm, 10 cm, 15 cm, 25 cm, 40 cm). Table 2 show the average values of the measured permeabilities.

Table 1 Distribution of aggregates dimensions for mix

| Diameter (mm) | 15 | 10 | 5 | 2 | 0.4 | 0.18 | 0.075 | Filler | Total |
|---|---|---|---|---|---|---|---|---|---|
| Basaltic 0/2 | | | | 0.8 | 8.0 | 2.4 | 1.9 | 4.9 | 18.0 |
| Basaltic 10/15 | 10.9 | 60.1 | 9.0 | | | | | | 80.0 |
| Filler | | | | | | | | 2.0 | 2.0 |

Table 2 Hydraulic permeability measured in laboratory

| Sample | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Bitumen (%) | 5 | | | | 5.5 | | | | 6 | | | |
| Density (g/cm$^3$) | 2.190 | 2.174 | 2.176 | 2.191 | 2.175 | 2.176 | 2.182 | 2.181 | 2.177 | 2.190 | 2.208 | 2.220 |
| Voids (%) | 15.82 | 16.44 | 16.35 | 15.76 | 15.76 | 15.75 | 15.52 | 15.53 | 15.08 | 14.59 | 13.90 | 13.42 |
| K (cm/s) | 0.033 | 0.083 | 0.043 | 0.011 | 0.021 | 0.026 | 0.015 | 0.053 | 0.038 | 0.022 | 0.027 | 0.011 |
| $\bar{K}$ (cm/s) | 0.042375 | | | | 0.0287 | | | | 0.024725 | | | |

### 3.4. Simulation Results

According to the HMA characteristics of the samples tested in laboratory, 12 synthetical samples are modelled. Four samples for each percentage of bitumen are simulated randomly starting from four different values of the initial point in the domain in which the algorithm inserts the aggregates.

For each of the 12 synthetical samples the model simulates the different flow paths of the water, as show in figure 5. The flow paths geometries allow to calculate the tortuosity, used in equation (9), by tracking a band of most probable water paths and by assuming an average trajectory to measure the length of the paths.

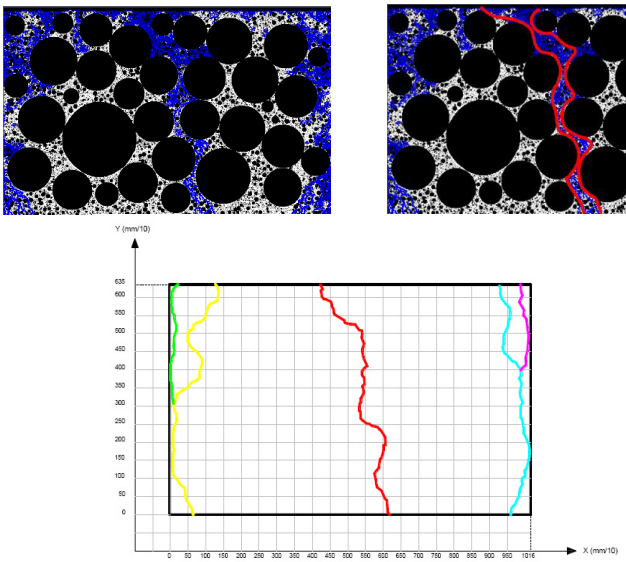


Fig. 5. (a), (b), (c) identification and measurement of the length of water paths

The calculation of the tortuosity values allow to predict the permeability coefficient (K) for each synthetic sample through equation (9). In particular we have defined, from the results of the simulation process, each parameters of the equation (9). The values of these parameters are shown in table 3.

Table 3 Hydraulic permeability of synthetic samples

| Sample | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Bitumen (%) | 5 | | | | 5.5 | | | | 6 | | | |
| $\gamma$ (g/cm$^3$) | 1 | | | | 1 | | | | 1 | | | |
| $\mu$ (g/cms) | $10^{-2}$ | | | | $10^{-2}$ | | | | $10^{-2}$ | | | |
| b | 2 | | | | 2 | | | | 2 | | | |
| $n_e$ | 0.269 | | | | 0.234 | | | | 0.212 | | | |
| $S_a$ (1/mm) | 3.932 | | | | 3.586 | | | | 3.351 | | | |
| T | 1.277 | 1.206 | 1.266 | 1.208 | 1.381 | 1.256 | 1.317 | 1.209 | 1.328 | 1.296 | 1.253 | 1.334 |
| K (cm/s) | 0.039 | 0.043 | 0.039 | 0.043 | 0.026 | 0.032 | 0.029 | 0.034 | 0.024 | 0.025 | 0.027 | 0.024 |
| K (cm/s) | 0.041075 | | | | 0.030125 | | | | 0.02505 | | | |

## 4. Discussion

Figure 6 compares the laboratory results (K-measured) with the theoretical permeability (K-predicted) calculated for each synthetical samples.

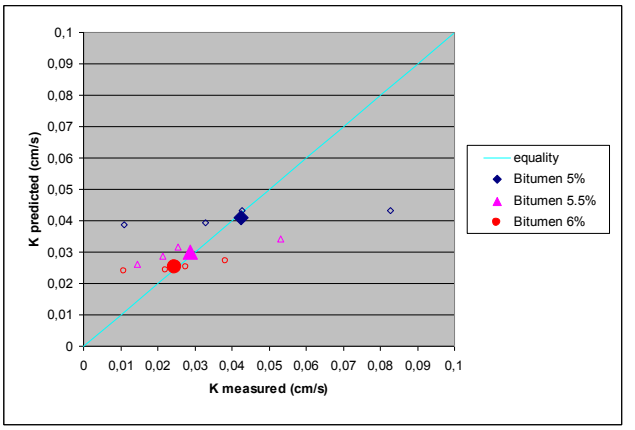


Fig. 6. comparison between laboratory results and theoretical permeability

Figure 6 shows a good correlation between the values of K measured and K predicted. In particular the average values of the three percentage of bitumen are positioned along the bisector. This can be reasonably considered an excellent result, assuming that the laboratory procedure, from the same initial conditions, produces different results, also under rigorous standards and methods. This is the reason why, in the laboratory tests, the average value is calculated over some tested samples in order to define the properties of the mix. In these conditions, because the numerical model is a promising approximation of the real cases, as shown in figure 6, it possible to extract more stable and representative averages from synthetical samples improving or substituting the laboratory test by a virtual approach.

## 5. Conclusion

The presence of free water within the asphalt pavement decreases the tire pavement friction and reduces the load carrying capacity through promoting distresses such as moisture cohesive and adhesive damage, fatigue cracking, and permanent deformation. In these conditions it is clear that understanding fluid transport characteristics of asphalt pavements is critical also in design life of these structures. There has been emphasis in the past few years on developing new approaches in order to evaluate the expected drainage capability of pavement during the mix design. However, these approaches consider simplified assumptions that bring to misleading results or they are too onerous by a computational and economical point of view. A numerical model is presented here for two-dimensional simulations of unsteady flow of water drainage through an open-graded asphalt mixes. The accuracy of the model is verified through experimental tests and analytical predictions. The experimental procedure relies on the use of a permeameter to measure Darcy permeability. The analytical predictions are based on theoretical equations validated in the literature. In these equations the permeability is inversely proportional to the square of a parameter that defines the tortuosity of flow paths. The values of tortuosity is estimated using the simulation developed in this study.

An excellent agreement is observed between the measures of laboratory tests and the results of the simulation. The model presented in this study can be applied for validation and design of open-graded asphalt mixtures.

## References

[1] Al-Omari, A., Tashman, L., Masad, E., Cooley, A., and Harman, T. (2002). Proposed Methodology for Predicting HMA Permeability. Journal of Association of Asphalt Paving Tech., 71, 30-57.
[2] Bear, J. (1972). Dynamics of fluids in porous media. American Elsevier Publishing Co, New York.
[3] Benedetto, A. (2007). Open graded asphalt mixes: permeability and mechanical characteristics. International Journal of Pavement, 6, 63-74.
[4] Benedetto, A. (2009). Numerical simulation of mechanical and hydraulic characteristics of open graded mix. Proceedings of Conference on Maintenance and Rehabilitation of Pavements and Technical Control Torino, Italy.
[5] Bhatnagar, P.L., Gross, E.P. and Krook, M. (1954). A Model for Collision Processes in Gases. I. Small Amplitude Processes in Charged and Neutral One-Component Systems. Physical Review, 94, 511–525.
[6] Carman, P.C. (1956). Flow of Gases through Porous Media. Academic Press, New York.
[7] Cercignani, C. (1975). Theory and Application of the Boltzmann Equation. Scottish Academic Press, 701120819.
[8] He, X. and Luo, L. (1997). Theory of Lattice Boltzmann Method: From the Boltzmann equation to the Lattice Boltzmann. Physical Review, 56, 6811-6817.
[9] Koplic, J., Lin, C., and Vermette, M. (1984). Conductivity and permeability from microgeometry. Journal of Applied Physics, 56, 3127-3131.
[10] Koponen, A., Kandhai, D., Hellen, E., Alava, M., Hoekstra, A., Kataja, M., Niskanen, K., Sloot, P. and Timonen, J. (1998). Permeability of Three Dimensional Random Fiber Webs. Physical Review, 80, 716-719.
[11] Kutay, M.E., Aydilek, A.H. and Masad, E. (2006). Laboratory validation of lattice Boltzmann method for modeling pore-scale flow in granular materials. Computers and Geotechnics, 33, 381-395.
[12] Lock, P.A., Jing, X D., Zimmerman, R.W. and Schlueter, E.M. (2002). Predicting the permeability of sandstone from image analysis of pore structure. Journal of Applied Physics, 92, 6311-6319.
[13] Maier, R., Kroll, D., Kutsovsky, Y., Davis, H. T. and Bernard, R. (1997). Simulation of Flow Through Bead Packs Using the Lattice Boltzmann Method. Physics of Fluids, 10, 60-75.
[14] Martys, N.S. and Hagedorn, J.G. (2002). Multiscale Modeling of Fluid Transport in Heteregenous Materials unisng Discrete Boltzmann Methods. Materials and Structures, 35, 650-659.
[15] Masad, E. (1998). Permeability simulation of anisotropic reconstructed soil medium. Ph.D. dissertation, Washington State University, Pullman, WA.
[16] McNamara, G. and Zanetti, G. (1988). Use of the Boltzmann equation to simulate lattice gas automata. Physical Review, 61, 2332-2335.
[17] Meakin, P. and Jullien, R. (1992). RSA of disks of different sizes. Physical Review , 46(4), 2029-2038.
[18] Metropolis, N. and Ulam, S. (1949). The Monte Carlo Method. Journal of American Statistical Association, 44, 335–341.
[19] Mualem, Y. (1976). A new model for predicting the hydraulic conductivity of unsaturated porous media. Water Resources Research, 12, 1248-1254.
[20] Rajani, B.B. (1988). A simple model for describing variation of permeability with porosity for unconsolidated sands. In Situ, 12(3), 209-226.
[21] Tashman, L., Masad, E., Crowe, C., and Muhunthan, B. (2003). Simulation of fluid flow in granular microstructure using a non-staggered grid scheme. Computers & Fluids, 32, 1299-1323.
[22] Taylor, S.W., Milly, P.C.D. and Jaffe, P. R. (1990). Biofilm growth and the related changes in the physical properties of a porous medium. 2. permeability. Water Resources Research, 26(9), 2161-2169.
[23] Von Neumann, J. (1940) . The estimation of probable error from successive differences. Aberdeen Proving Ground, Report 175, 19 pp.
[24] Walsh, J.B. and Brace, W.F. (1984). The Effect of Pressure in Porosity and the Transport Properties of Rock. Journal of Geophysical Research, 89, 9425-9431.
[25] Wang, J. C., Leung, C. F., and Chow, Y. K. (2003). Numerical solutions for flow in porous media. International Journal for Numerical and Analytical Methods in Geomechanics, 27, 565 - 583.